FEYNMAN DIAGRAMS FOR ABA THEORY

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 19.3.

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In ABA theory, the Lagrangian is

$$\mathcal{L}=\frac{1}{2}\left(\partial_{\mu}\phi_{A}\right)^{2}-\frac{m_{A}^{2}}{2}\phi_{A}^{2}+\frac{1}{2}\left(\partial_{\mu}\phi_{B}\right)^{2}-\frac{m_{B}^{2}}{2}\phi_{B}^{2}-\frac{g}{2}\phi_{A}\phi_{B}\phi_{A} \tag{1}$$

where ϕ_A and ϕ_B are two different scalar fields. Because the fields are different, they commute with each other. The interaction term is $\frac{g}{2}\phi_A\phi_B\phi_A$, so the interaction vertex contains 2 lines for A particles and one line for a B particle. If we draw A particles with black lines and B particles with red lines, the interaction vertex is in Fig. 1.

As with our earlier example of ϕ^3 theory, because the interaction contains an odd number of lines, only even-order terms appear in the scattering amplitude. Thus there are no contributions from first- or third-order terms.

Note that if we apply Wick's theorem to a term involving products of field operators of different types, as we have here, a contraction of two different types of operator (as with ϕ_A and ϕ_B) gives zero, so there are no 'hybrid lines' consisting of a combination of A and B type particles. This is because a contraction is given by

$$\left\langle 0\left|\mathscr{T}\left[\psi_{\alpha}(x)\,\psi_{\beta}\left(x'\right)\right]\right|0\right\rangle \equiv \contraction{}{\psi}{_{\alpha}(x)}{\psi}\psi_{\alpha}(x)\,\psi_{\beta}\left(x'\right) \tag{2}$$

If ψ_α and ψ_β represent different types of particle, then ψ_α, say, creates a particle of type α and ψ_β annihilates a particle of type β. However, if ψ_β

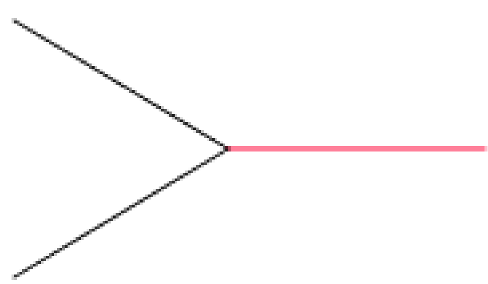

FIGURE 1. An interaction vertex.

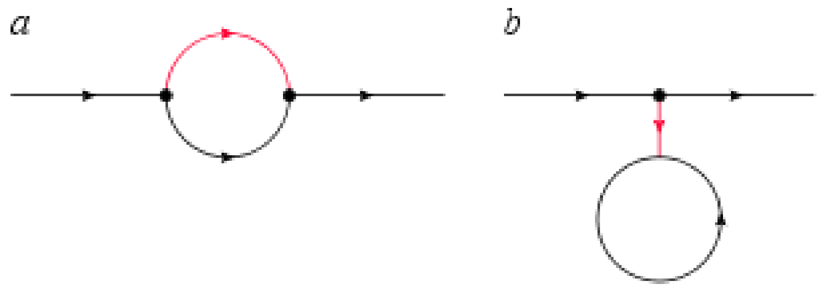

FIGURE 2. Second-order diagrams for $\langle q_A|S|p_A\rangle$

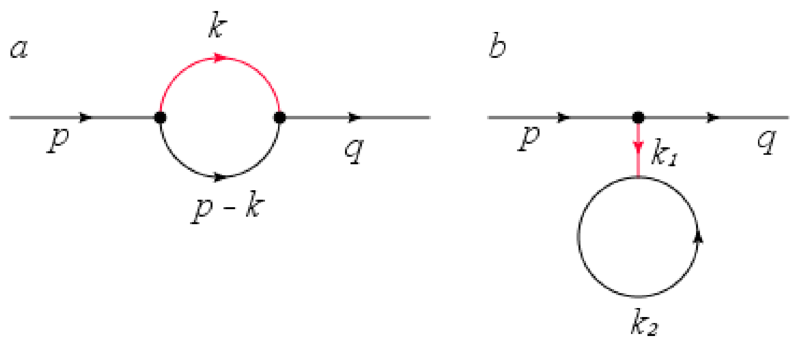

FIGURE 3. Second-order diagrams for $\langle q_A|S|p_A\rangle$ with momenta.

operates after ψ_α, there are no type-β particles to annihilate, so operating with ψ_β gives zero. In general, a field operator is a sum of a creation part and an annihilation part, so we can apply this argument to the creation part of one operator and the annihilation part of the other to see that the Wick contraction of two different types of field will always yield zero.

We look first at single-particle terms. The allowable diagrams form a subset of those diagrams that we used for ϕ^3 theory, since in the former theory, all lines referred to the same type of particle, while here we must have two A particles and one B particle joining at each vertex. If we consider the scattering amplitude $\langle q_A|S|p_A\rangle$, where an A particle enters with momentum p_A and leaves with momentum q_A, then the external lines must both be of type A (black). The second-order diagrams are shown in Fig. 2.

To get the amplitudes in momentum space, we label the momenta on each branch, as in Fig. 3. We can use rules similar to those for ϕ^4 theory to write the amplitudes.

In each case, we have a factor of $-ig$ for each vertex and a factor of $\frac{i}{k^2-m_j^2+i\epsilon}$ for each internal line, where the suffix j on m_j refers to either A or B. We apply momentum conservation at each vertex, and integrate over unconstrained momenta with an integration measure of $\frac{d^4k}{(2\pi)^4}$. Then we divide by the symmetry factor D, and finally, we multiply by a factor of $(2\pi)^4\,\delta^{(4)}\,(p-q)$ to conserve overall momentum.

For (a) in Fig. 3, these rules give us an amplitude of

$$(2\pi)^4\,\delta^{(4)}\,(p-q)\,\frac{(-ig)^2}{D}\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m_B^2+i\epsilon}\frac{i}{(p-k)^2-m_A^2+i\epsilon} \quad (3)$$

In diagram (b), momentum conservation at the top vertex requires that $p=q$ and thus $k_1=0$. Thus this diagram contributes nothing to the amplitude.

We now consider fourth-order diagrams for $\langle q_A|S|p_A\rangle$. Again, these are a subset of the diagrams for ϕ^3 theory subject to the constraint that each vertex must be of the form given in Fig. 1. The valid diagrams are in Fig. 4.

In ϕ^3 theory, there was one additional fourth-order diagram, but this cannot be converted into a valid diagram in ABA theory because we can't satisfy the constraint at each vertex. This diagram is shown in Fig. 5.

Because we must have an A particle as the incoming and outgoing particle, both lines at the top must be black. This forces the downward path to be red, which in turn forces the two diagonal paths emerging from it to be black. However, since the loops at the ends of these two black paths are each a single path, they can't satisfy the constraint that we have two black and one red path converging on the vertex at the end of each diagonal line. The loops are shown in blue to show that this is impossible.

We can now consider the diagrams for the amplitude $\langle q_B|S|p_B\rangle$. In this case, the incoming and outgoing lines must be red, and we apply the interaction vertex constraint to all internal vertices. We have Fig. 6.

We can't have a diagram like Fig. 2(b) since that would require the two horizontal paths at the top to both be red, which violates the constraint.

For the two-particle scattering amplitude $\langle q_{A1}q_{A2}|S|p_{A2}p_{A1}\rangle$, we can again model our diagrams after those in ϕ^3 theory with the additional constraint. There is one second-order diagram, shown in Fig. 7.

At fourth-order, we have the diagrams shown in Fig. 8.

The symmetry factors for these diagrams can be determined by considering what permutations of the internal paths can be made without altering the topology of the diagram. It would seem that we can permute internal lines that join the same two vertices, just as in ϕ^3 theory. For example, in Fig. 3(a), if we swap the two internal arcs that would result in the amplitude 3 we get

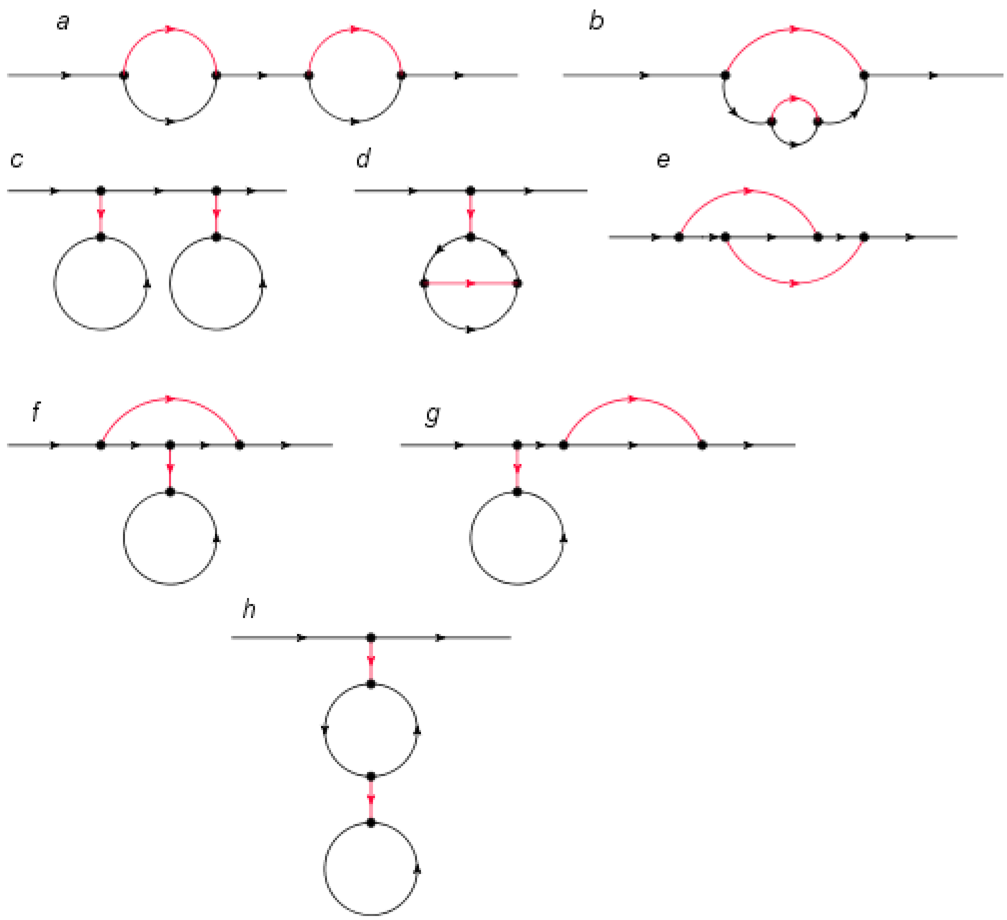

FIGURE 4. Fourth-order diagrams for $\langle q_A|S|p_A\rangle$

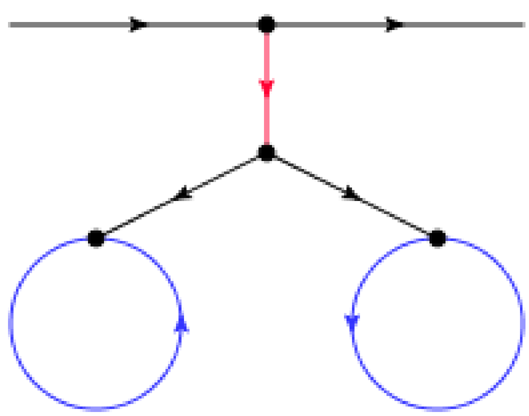

FIGURE 5. Invalid fourth-order diagram.

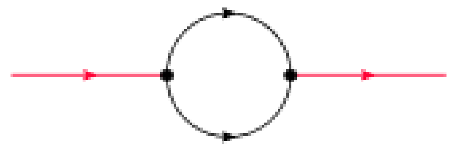

FIGURE 6. Second-order diagram for $\langle q_B | S | p_B \rangle$

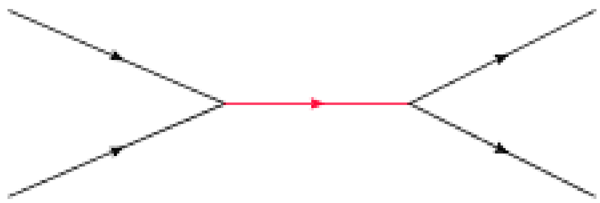

FIGURE 7. Second-order scattering diagram.

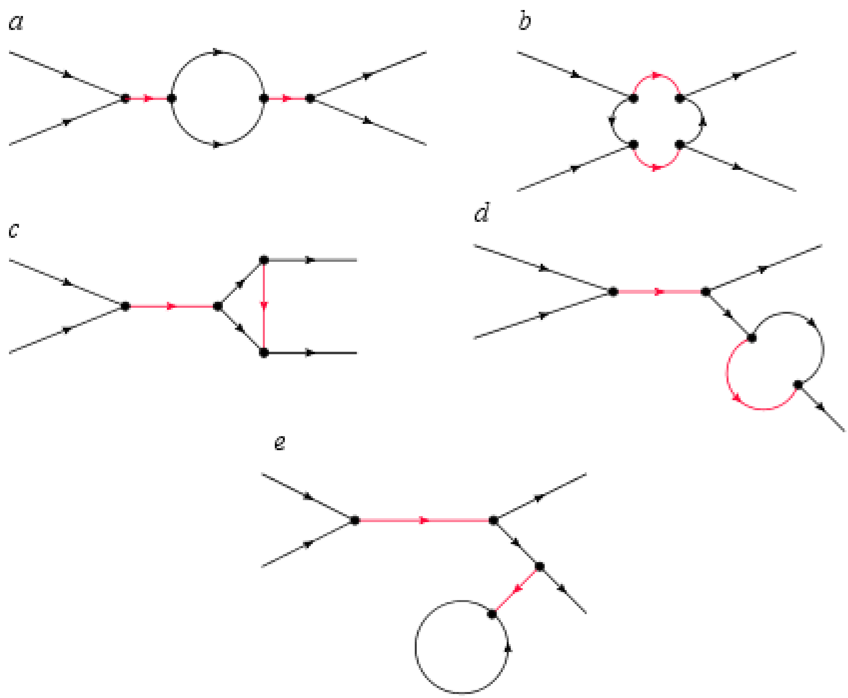

FIGURE 8. Fourth-order scattering diagrams.

$$(2\pi)^4 \delta^{(4)}(p-q) \frac{(-ig)^2}{D} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m_A^2 + i\epsilon} \frac{i}{(p-k)^2 - m_B^2 + i\epsilon} \quad (4)$$

However, if we make the substitution $k' = p - k$, this integral becomes the same as the original 3.

PINGBACKS

Pingback: Yukawa's psi-psi-phi theory