YUKAWA'S PSI-PSI-PHI THEORY

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 20.1.

Post date: 23 Jul 2019.

Yukawa's $\psi^{\dagger}\psi\phi$ theory applies to a system with Lagrangian

$$\mathcal{L}=\partial^{\mu}\psi^{\dagger}\partial_{\mu}\psi-m^{2}\psi^{\dagger}\psi+\frac{1}{2}\left(\partial_{\mu}\phi\right)^{2}-\frac{1}{2}\mu^{2}\phi^{2}-g\psi^{\dagger}\psi\phi \tag{1}$$

The last term is the interaction term. This theory is similar to the ABA theory we considered earlier in that we have an interaction vertex involving three particles, with two of one type and one of a different type. As in that case, the expansion of the S-matrix leads to non-zero terms only for even order, and contractions between $\psi^{\dagger}$ or ψ and ϕ are zero. The non-zero contractions are given in L&B's equation 20.6, and we're asked to verify them. This amounts to checking these contractions with their original derivations in equations 17.24 and 19.14. Equation 17.24 gives us an expression for the free propagator, which is equivalent to a contraction. We have

$$\psi(x)\psi^{\dagger}(y)=\left\langle 0\left|T\psi(x)\psi^{\dagger}(y)\right|0\right\rangle \tag{2}$$

$$=\int\frac{d^{4}p}{(2\pi)^{4}}\frac{ie^{-ip\cdot(x-y)}}{p^{2}-m^{2}+i\epsilon} \tag{3}$$

The middle term in quation 17.24 should read $\left\langle 0\left|T\phi(x)\phi^{\dagger}(y)\right|0\right\rangle$ (with a † on the second ϕ) since it is a rewriting of equation 17.15.

For the real $\phi(x)$ field, we rewrite this by replacing the mass m by μ and using a different integration variable q to get

$$\phi(x)\phi(y)=\langle 0|T\phi(x)\phi(y)|0\rangle \tag{4}$$

$$=\int\frac{d^{4}q}{(2\pi)^{4}}\frac{ie^{-iq\cdot(x-y)}}{q^{2}-\mu^{2}+i\epsilon} \tag{5}$$

The other contractions in equation 20.6 all involve contracting a field operator with a creation or annihilation operator, and this is derived in equation 19.14, which is

$$\phi(x)a_{\boldsymbol{p}}^{\dagger}=\frac{1}{(2\pi)^{\frac{3}{2}}\left(2E_{\boldsymbol{p}}\right)^{\frac{1}{2}}}e^{-ip\cdot x} \tag{6}$$

This result is derived by expressing the field as a Fourier integral over the creation and annihilation operators:

$$\phi(x) = \int \frac{d^3 q}{(2\pi)^{\frac{3}{2}} (2E_{\boldsymbol{q}})^{\frac{1}{2}}} \left(a_{\boldsymbol{q}} e^{-iq\cdot x} + a_{\boldsymbol{q}}^\dagger e^{iq\cdot x} \right) \tag{7}$$

We then calculate the vacuum expectation value $\left\langle 0 \middle| \phi(x) a_{\boldsymbol{p}}^\dagger \middle| 0 \right\rangle$ using this expansion to get 6.

In the Yukawa $\psi^\dagger \psi \phi$ theory we have the expansions given in Equation 20.2:

$$\psi(x) = \int \frac{d^3 p}{(2\pi)^{\frac{3}{2}} (2E_{\boldsymbol{p}})^{\frac{1}{2}}} \left(a_{\boldsymbol{p}} e^{-ip\cdot x} + b_{\boldsymbol{p}}^\dagger e^{ip\cdot x} \right) \tag{8}$$

$$\psi^\dagger(x) = \int \frac{d^3 p}{(2\pi)^{\frac{3}{2}} (2E_{\boldsymbol{p}})^{\frac{1}{2}}} \left(a_{\boldsymbol{p}}^\dagger e^{ip\cdot x} + b_{\boldsymbol{p}} e^{-ip\cdot x} \right) \tag{9}$$

$$\phi(x) = \int \frac{d^3 q}{(2\pi)^{\frac{3}{2}} (2\varepsilon_{\boldsymbol{q}})^{\frac{1}{2}}} \left(c_{\boldsymbol{q}} e^{-iq\cdot x} + c_{\boldsymbol{q}}^\dagger e^{iq\cdot x} \right) \tag{10}$$

where

$$E_{\boldsymbol{p}} = \sqrt{\boldsymbol{p}^2 + m^2} \tag{11}$$

$$\varepsilon_{\boldsymbol{q}} = \sqrt{\boldsymbol{q}^2 + \mu^2} \tag{12}$$

From 6 and 7, we see that a contraction of a field operator with a creation operator $\underbrace{\phi(x)\, a_{\boldsymbol{p}}^\dagger}$ results in the exponential factor $e^{-iq\cdot x}$ attached to the annihilation operator $a_{\boldsymbol{q}}$ in the Fourier expansion. For a contraction of the conjugate field operater with an annihilation operator, we take the exponential factor attached to the creation operator. This gives the remaining six contractions in Equation 20.6:

$$\underbracket{a_{\boldsymbol{p}}\psi^{\dagger}}\left(x\right)=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}\left(2E_{\boldsymbol{p}}\right)^{\frac{1}{2}}}e^{ip\cdot x} \tag{13}$$

$$\underbracket{b_{\boldsymbol{p}}\psi}\left(x\right)=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}\left(2E_{\boldsymbol{p}}\right)^{\frac{1}{2}}}e^{ip\cdot x} \tag{14}$$

$$\underbracket{c_{\boldsymbol{q}}\phi}\left(x\right)=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}\left(2\varepsilon_{\boldsymbol{q}}\right)^{\frac{1}{2}}}e^{iq\cdot x} \tag{15}$$

$$\underbracket{\psi\left(x\right)a_{\boldsymbol{p}}^{\dagger}}=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}\left(2E_{\boldsymbol{p}}\right)^{\frac{1}{2}}}e^{-ip\cdot x} \tag{16}$$

$$\underbracket{\psi^{\dagger}\left(x\right)b_{\boldsymbol{p}}^{\dagger}}=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}\left(2E_{\boldsymbol{p}}\right)^{\frac{1}{2}}}e^{-ip\cdot x} \tag{17}$$

$$\underbracket{\phi\left(x\right)c_{\boldsymbol{q}}^{\dagger}}=\frac{1}{\left(2\pi\right)^{\frac{3}{2}}\left(2\varepsilon_{\boldsymbol{q}}\right)^{\frac{1}{2}}}e^{-iq\cdot x} \tag{18}$$

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