LAGRANGIAN FOR GAUSSIAN INTEGRAL

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 23.1.

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This is an odd little problem since I'm not sure what its relevance to anything else is. We start with a Lagrangian

$$L=\frac{1}{2}xAx+bx \tag{1}$$

where A is an operator and b is a constant. Using the Euler-Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{x}}=\frac{\partial L}{\partial x} \tag{2}$$

we have

$$\frac{\partial L}{\partial\dot{x}}=0 \tag{3}$$

$$\frac{\partial L}{\partial x}=\frac{1}{2}Ax+\frac{1}{2}xA+b \tag{4}$$

so we have (treating A as a number):

$$x=-\frac{b}{A} \tag{5}$$

Therefore the Lagrangian 1 can be written as

$$L=\frac{1}{2}\left(-\frac{b}{A}\right)A\left(-\frac{b}{A}\right)-\frac{b^2}{A} \tag{6}$$

$$=-\frac{b^2}{2A} \tag{7}$$

$$=-b\frac{1}{2A}b \tag{8}$$

The closest thing to this in Chaper 23 seems to be equation 23.16, which is the Gaussian integral

$$\int_{-\infty}^{\infty} dx\, e^{-\frac{ax^2}{2}+bx} = \sqrt{\frac{2\pi}{a}} e^{\frac{b^2}{2a}} \tag{9}$$

However, one exponent is an integrand and the other is a stand-alone result. Also, the signs don't match up, since we have $-\frac{ax^2}{2}+bx$ in the integral versus $+\frac{1}{2}xAx+bx$ in the Lagrangian. If anyone can see the point of this exercise, please do leave a comment.