# PROPAGATOR FOR FREE MASSIVE VECTOR FIELD

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 24.1.

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We consider the Lagrangian density for a free massive vector field, which is given by L&B's equation 24.23:

$$\mathcal{L}=-\frac{1}{2}\left(\partial_{\mu}A_{\nu}\partial^{\mu}A^{\nu}-\partial_{\mu}A_{\nu}\partial^{\nu}A^{\mu}\right)+\frac{1}{2}m^{2}A^{\mu}A_{\mu} \tag{1}$$

In order to get the generating functional, we need the Lagrangian in the form $\frac{1}{2}A^{\mu}K_{\mu\nu}A^{\nu}$ where $K_{\mu\nu}$ is some operator. In fact, we've already done this earlier by using the trick of adding a total divergence to the Lagrangian density. Since it's the integral of $\mathcal{L}$ over all space that we want, the integral of a total divergence converts to a surface integral by Gauss's theorem, and we make the usual assumption that the fields all go to zero fast enough for this surface integral to vanish at infinity. The result from the earlier post is

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}A^{\mu}\left(\partial^{2}A_{\mu}-\partial_{\mu}\partial^{\nu}A_{\nu}\right)+\frac{1}{2}m^{2}A^{\mu}A_{\mu} && (2)\\
&= \frac{1}{2}A^{\mu}\left(g_{\mu\nu}\partial^{2}A^{\nu}-\partial_{\mu}\partial_{\nu}A^{\nu}+m^{2}g_{\mu\nu}A^{\nu}\right) && (3)\\
&= \frac{1}{2}A^{\mu}\left[g_{\mu\nu}\left(\partial^{2}+m^{2}\right)-\partial_{\mu}\partial_{\nu}\right]A^{\nu} && (4)
\end{aligned}$$

Thus the operator is given by

$$K_{\mu\nu}=g_{\mu\nu}\left(\partial^{2}+m^{2}\right)-\partial_{\mu}\partial_{\nu} \tag{5}$$

In K&B's equation 24.28, the give an equation for the Fourier transform of the propagator, which is

$$\left[-\left(p^{2}+m^{2}\right)g^{\mu\nu}+p^{\mu}p^{\nu}\right]\tilde{G}_{0\nu\lambda}(p)=ig_{\lambda}^{\mu} \tag{6}$$

The solution is given as equation 24.29:

$$\tilde{G}_{0\nu\lambda}(p)=\frac{-i\left(g_{\nu\lambda}-p_{\nu}p_{\lambda}/m^{2}\right)}{p^{2}-m^{2}} \tag{7}$$

We can verify this is a solution of 6 by direct substitution. We have

$$\left[-\left(p^2+m^2\right)g^{\mu\nu}+p^\mu p^\nu\right]\frac{-i\left(g_{\nu\lambda}-p_\nu p_\lambda/m^2\right)}{p^2-m^2}=ig^{\mu\nu}g_{\nu\lambda}-ig^{\mu\nu}\frac{p_\nu p_\lambda}{m^2}-\frac{ig_{\nu\lambda}p^\mu p^\nu}{p^2-m^2}+\frac{ip^\mu p^\nu p_\nu p_\lambda}{m^2\left(p^2-m^2\right)} \tag{8}$$

$$=ig^\mu_\lambda-ig^{\mu\nu}\frac{p_\nu p_\lambda}{m^2}-\frac{ig_{\nu\lambda}p^\mu p^\nu}{p^2-m^2}+ip^2p^\mu p_\lambda\frac{1}{p^2}\left[\frac{1}{m^2}+\frac{1}{p^2-m^2}\right] \tag{9}$$

$$=ig^\mu_\lambda-i\frac{p^\mu p_\lambda}{m^2}-\frac{ip^\mu p_\lambda}{p^2-m^2}+\frac{ip^\mu p_\lambda}{m^2}+\frac{ip^\mu p_\lambda}{p^2-m^2} \tag{10}$$

$$=ig^\mu_\lambda \tag{11}$$