CONNECTED CORRELATION FUNCTION IN FIELD THEORY

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 25.2.

Post date: 15 Sep 2019.

Given the partition function in field theory (L&B's equation 25.26) we need to calculate the connected correlation function. The partition function is

$$Z[J]=\int\mathcal{D}\phi e^{-\int d\tau d^{3}x\left(\mathcal{L}_{E}[\phi]-J\phi\right)} \tag{1}$$

where $\mathcal{L}_E$ is the lagrangian in Euclidean space and J is the source term. [See L&B chapter 25 for a discussion of the Wick rotation which leads to Euclidean space.]

If we start with $\ln Z[J]$ and take its derivative, we have

$$\frac{\partial\ln Z[J]}{\partial J(x)} = \frac{1}{Z}\frac{\partial Z}{\partial J(x)} \tag{2}$$

$$= \frac{1}{Z}\int\mathcal{D}\phi\,\phi(x)\,e^{-\int d\tau d^{3}x\left(\mathcal{L}_{E}[\phi]-J\phi\right)} \tag{3}$$

$$\frac{\partial^{2}\ln Z[J]}{\partial J(x)\,\partial J(y)} = -\frac{1}{Z^{2}}\frac{\partial Z}{\partial J(y)}\frac{\partial Z}{\partial J(x)}+\frac{1}{Z}\frac{\partial^{2}Z}{\partial J(x)\,J(y)} \tag{4}$$

$$= -\frac{1}{Z^{2}}\int\mathcal{D}\phi\,\phi(y)\,e^{-\int d\tau d^{3}x\left(\mathcal{L}_{E}[\phi]-J\phi\right)}\times$$
$$\int\mathcal{D}\phi\,\phi(x)\,e^{-\int d\tau d^{3}x\left(\mathcal{L}_{E}[\phi]-J\phi\right)}+$$
$$\frac{1}{Z}\int\mathcal{D}\phi\,\phi(x)\,\phi(y)\,e^{-\int d\tau d^{3}x\left(\mathcal{L}_{E}[\phi]-J\phi\right)} \tag{5}$$

If we now set $J=0$ and use the definition of the thermal expectation value of a field (see L&B, Chapter 21), which is

$$\left\langle \phi(x)\right\rangle_{t} = \frac{1}{Z}\int\mathcal{D}\phi\,\phi(x)\,e^{-\int d\tau d^{3}x\left(\mathcal{L}_{E}[\phi]-J\phi\right)}\Bigg|_{J=0} \tag{6}$$

then from the above we have

$$\left.\frac{\partial^2 \ln Z[J]}{\partial J(x)\,\partial J(y)}\right|_{J=0} = -\langle\phi(x)\rangle_t\langle\phi(y)\rangle_t + \langle\phi(x)\,\phi(y)\rangle_t \equiv G_c(x,y) \quad (7)$$