INTERNAL SYMMETRY OF THE VACUUM

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 26.1.

Post date: 21 Sep 2019.

We consider an internal symmetry of the vacuum state $|0\rangle$. We have two types of particles, A and B, with creation operators $\phi_A^\dagger$ and $\phi_B^\dagger$. We now consider some operator (a generator of a symmetry) Q_N that satisfies the commutation relations

$$\left[Q_N,\phi_A^\dagger\right]=\phi_B^\dagger \tag{1}$$

$$[Q_N,H]=0 \tag{2}$$

We also assume that Q_N annihilates the vacuum, so that

$$Q_N|0\rangle=0 \tag{3}$$

In this case, we have

$$e^{i\alpha Q_N}|0\rangle=e^{i\alpha 0}|0\rangle=|0\rangle \tag{4}$$

Thus the vacuum state $|0\rangle$ is invariant under the internal symmetry transformation $e^{i\alpha Q_N}|0\rangle$.

Now suppose that the hamiltonian H has two eigenstates, one of which contains a single particle of type A and the other of type B. That is

$$H\phi_A^\dagger|0\rangle=E_A\phi_A^\dagger|0\rangle \tag{5}$$

$$H\phi_B^\dagger|0\rangle=E_B\phi_B^\dagger|0\rangle \tag{6}$$

If we operate on the LHS of 5 with Q_N on the left, we have, using the above relations:

$$Q_N H\phi_A^\dagger|0\rangle = HQ_N\phi_A^\dagger|0\rangle \tag{7}$$
$$= H\left(\phi_B^\dagger + \phi_A^\dagger Q_N\right)|0\rangle \tag{8}$$
$$= H\phi_B^\dagger|0\rangle + 0 \tag{9}$$
$$= E_B\phi_B^\dagger|0\rangle \tag{10}$$

Operating on the RHS of 5 with Q_N we have

$$Q_N E_A\phi_A^\dagger|0\rangle = E_A Q_N\phi_A^\dagger|0\rangle \tag{11}$$
$$= E_A\left(\phi_B^\dagger + \phi_A^\dagger Q_N\right)|0\rangle \tag{12}$$
$$= E_A\phi_B^\dagger|0\rangle + 0 \tag{13}$$
$$= E_A\phi_B^\dagger|0\rangle \tag{14}$$

Comparing 10 and 14 we see that

$$E_A = E_B \tag{15}$$

Thus if the vacuum is invariant under the internal symmetry 4, both fields have the same energy. This relies on 3, however, so that if Q_N didn't annihilate the vacuum, then the fields would have different energies.