FABRI-PICASSO THEOREM

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 26.2.

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The Noether charge is given in terms of the 0 component of the Noether current by

$$Q_N = \int d^3x\ J_N^0(x) \tag{1}$$

The translation operator for translation by a distance a is $e^{ip\cdot a}$ so

$$J_N^0(x) = e^{ip\cdot x} J_N^0(0)\, e^{-ip\cdot x} \tag{2}$$

The vacuum state expectation value in the question is then

$$\left\langle 0\left|J_N^0(x)\,Q_N\right|0\right\rangle = \left\langle 0\left|e^{ip\cdot x} J_N^0(0)\, e^{-ip\cdot x} Q_N\right|0\right\rangle \tag{3}$$

Since the vacuum state has zero energy and momentum, the momentum operator p acting on the vacuum state gives 0, so the RHS of this equation results in

$$\left\langle 0\left|J_N^0(x)\,Q_N\right|0\right\rangle = \left\langle 0\left|e^0 J_N^0(0)\, e^0 Q_N\right|0\right\rangle \tag{4}$$

$$= \left\langle 0\left|J_N^0(0)\,Q_N\right|0\right\rangle \tag{5}$$

Now consider the matrix element

$$\left\langle 0\left|Q_N Q_N\right|0\right\rangle = \left\langle 0\left|\left(\int d^3x\ J_N^0(x)\right) Q_N\right|0\right\rangle \tag{6}$$

Since the only place where x appears is the $J_N^0(x)$ term in the integrand, we can take the integral sign outside the matrix element to get

$$\left\langle 0\left|Q_N Q_N\right|0\right\rangle = \int d^3x \left\langle 0\left|J_N^0(x)\,Q_N\right|0\right\rangle \tag{7}$$

$$= \int d^3x \left\langle 0\left|J_N^0(0)\,Q_N\right|0\right\rangle \tag{8}$$

where we used 5 to get the last line. However, the integrand $\left\langle 0\left|J_N^0(0)\,Q_N\right|0\right\rangle$ is now independent of x, so can be treated as a constant with respect to the integral. If this constant is zero, then the integral is zero, but if the constant is not zero, then the integral is infinite, as we're integrating a non-zero constant over infinite space. Thus the state $Q_N |0\rangle$ must have a norm of either zero or infinity.