COHERENT STATES: BALANCED UNCERTAINTY

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 27.5.

Post date: 8 Oct 2019

A coherent state of the harmonic oscillator is a state $|\alpha\rangle$ that is an eigenstate of the operator

$$\hat{\alpha}=\frac{1}{\sqrt{2}}\left(\hat{Q}+i\hat{P}\right) \tag{1}$$

where $\hat{Q}$ is the position operator and $\hat{P}$ is the momentum operator. That is, we have

$$\frac{1}{\sqrt{2}}\left(\hat{Q}+i\hat{P}\right)|\alpha\rangle=\alpha|\alpha\rangle \tag{2}$$

where α is the (complex) eigenvalue.

We have the results for the expectation values

$$\langle Q\rangle=\sqrt{2}\Re\alpha \tag{3}$$

$$\langle P\rangle=\sqrt{2}\Im\alpha \tag{4}$$

so we have

$$(\langle Q\rangle+i\langle P\rangle)|\alpha\rangle=\sqrt{2}(\Re\alpha+i\Im\alpha)|\alpha\rangle \tag{5}$$

$$=\sqrt{2}\alpha|\alpha\rangle \tag{6}$$

Combining this with 2 we have

$$\left(\hat{Q}+i\hat{P}\right)|\alpha\rangle=(\langle Q\rangle+i\langle P\rangle)|\alpha\rangle \tag{7}$$

or

$$\left(\hat{Q}-\langle Q\rangle\right)|\alpha\rangle=-i\left(\hat{P}-\langle P\rangle\right)|\alpha\rangle \tag{8}$$

so that the uncertainty is balanced between the position and momentum.