COHERENT STATES: DIRAC'S PHASE OPERATOR

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 27.6.

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In the coherent state of the harmonic oscillator, Dirac suggested that a phase operator $\hat{\phi}$ could be defined via

$$\hat{a} = e^{i\hat{\phi}}\sqrt{\hat{n}} \tag{1}$$

where $\hat{n}$ is the usual number operator

$$\hat{n} = \hat{a}^{\dagger}\hat{a} \tag{2}$$

It's not obvious how to interpret the square root of the number operator, and it appears that Dirac abandoned this line of enquiry.

We can rearrange the definition to give

$$e^{i\hat{\phi}} = \hat{a}\left(\hat{n}\right)^{-1/2} \tag{3}$$

and use this to calculate the commutator (I'll drop the hats on the operators to save typing):

$$\left[\hat{n}, e^{i\hat{\phi}}\right] = a^{\dagger}aan^{-1/2} - an^{-1/2}a^{\dagger}a \tag{4}$$

Since $n = a^{\dagger}a$, n commutes with the combination $a^{\dagger}a$, so we have, using $\left[a, a^{\dagger}\right] = 1$:

$$\left[\hat{n}, e^{i\hat{\phi}}\right] = a^{\dagger}aan^{-1/2} - aa^{\dagger}an^{-1/2} \tag{5}$$

$$= -\left[a, a^{\dagger}\right]an^{-1/2} \tag{6}$$

$$= -an^{-1/2} \tag{7}$$

$$= -e^{i\phi} \tag{8}$$