TIDAL ACCELERATION IN NEWTONIAN PHYSICS

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Reference: Charles W. Misner, Kip S. Thorne & John Archibald Wheeler, *Gravitation*, W.H. Freeman (1973). Section 1.6.

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In Section 1.6, MTW derive the equations for tidal acceleration in Newtonian physics in their equation 1.5. Although they provide a proof, it might be useful to expand on it a bit to get a clearer understanding. (We've looked at this before from a slightly more advanced standpoint. The treatment here is a bit more basic.)

Suppose we are a distance r from the centre of a planet (like Earth) with mass m. Then our acceleration due to gravity is given by Newton's gravitational force formula and is

$$\frac{d^2r}{dt^2} = -\frac{Gm}{r^2} \tag{1}$$

(negative because we're accelerating towards the planet, so r is decreasing). Now suppose there is another observer at the same distance r from the planet but a small distance ξ away from us. The situation is as shown in Fig. 1.

In the figure, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the vector positions of the two observers and satisfy

$$|\boldsymbol{r}_1| = |\boldsymbol{r}_2| = r \tag{2}$$

That is, the vectors have the same length. For a small separation of the two observers (that is, ξ is small), the angle α is approximately

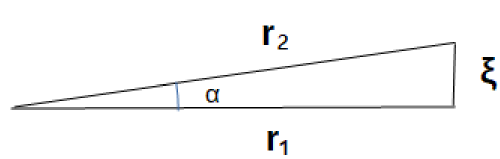

FIGURE 1. Two observers a distance r from the centre of a mass m.

$$\alpha = \frac{\xi}{r} \tag{3}$$

or, rearranging,

$$\xi = \alpha r \tag{4}$$

If we multiply 1 on both sides by α (which is a constant, since the two observers both accelerate towards the planet along a straight line path), we have

$$\begin{aligned} \alpha \frac{d^2 r}{dt^2} &= -\frac{Gm}{r^2}\alpha \\ \frac{d^2 \xi}{dt^2} &= -\frac{Gm}{r^3}\xi \end{aligned} \tag{5}$$

This equation is valid for any two observers situated at the same distance from the planet, and separated by a small distance. Thus this results shows the first two equations in MTW's equation 1.5.

For the third equation, suppose now that the two observers lie along the same straight line path from the planet, and one observer is at a distance r with the second observer at a distance $r+\xi$. The acceleration of the first observer is now given by 1 with $r = r_1$:

$$\frac{d^2 r_1}{dt^2} = -\frac{Gm}{r^2} \tag{6}$$

. For the second observer, we have, for small ξ

$$\frac{d^2 r_2}{dt^2} = -\frac{Gm}{(r+\xi)^2} \tag{7}$$

$$= -\frac{Gm}{r^2}\frac{1}{\left(1+\frac{\xi}{r}\right)^2} \tag{8}$$

$$\approx -\frac{Gm}{r^2}\left(1-\frac{2\xi}{r}\right) \tag{9}$$

$$= -\frac{Gm}{r^2} + \frac{2Gm\xi}{r^3} \tag{10}$$

Since the distance ξ between the two observers is $\xi = r_2 - r_1$, the relative acceleration of the two observers is now

$$\frac{d^2\xi}{dt^2} = \frac{d^2r_2}{dt^2} - \frac{d^2r_1}{dt^2} \tag{11}$$

$$= \frac{2Gm\xi}{r^3} \tag{12}$$

which is the third equation in MTW's equation 1.5.

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