GEODESIC DEVIATION ON A CYLINDER

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Reference: Charles W. Misner, Kip S. Thorne & John Archibald Wheeler, *Gravitation*, W.H. Freeman (1973). Exercise 1.1.

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Geodesic deviation is a measure of how the distance between two geodesics, initially parallel, varies as we move along the geodesic curves. On a sphere of radius a, we've seen that the geodesic deviation ξ obeys the simple harmonic equation

$$\frac{d^2\xi}{ds^2}+\frac{1}{a^2}\xi=0 \tag{1}$$

where s is the arc length we move along either geodesic. In this case, the Gaussian curvature R is

$$R=\frac{1}{a^2} \tag{2}$$

and the two principal radii of curvature are the same

$$\rho_1=\rho_2=a \tag{3}$$

On a cylinder, we can find a geodesic (shortest distance between two points on the surface of the cylinder) by cutting the cylinder along a line parallel to its axis and unrolling it. We then find that the surface of the cylinder unrolls into a plane, so a geodesic is the straight line between the two points. Since the cylinder unrolls into a plane, it has zero Gaussian curvature.

Another way of looking at it is to consider the cylinder in its original (rolled up) form. For a point on the surface, one principal radius of curvature is the radius (call it r) of the circle that results from a cross-section of the cylinder. In the perpendicular direction, however (that is parallel to the axis), a line along the cylinder is a straight line, which has an infinite radius of curvature. That is, the two principal radii are

$$\rho_1=a \tag{4}$$

$$\rho_2=\infty \tag{5}$$

The Gaussian curvature in this case is given by

$$R = \frac{1}{\rho_1 \rho_2} = 0 \tag{6}$$