KEPLER DENSITY

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Reference: Charles W. Misner, Kip S. Thorne & John Archibald Wheeler, *Gravitation*, W.H. Freeman (1973). Exercise 1.3.

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We're given a satellite in a circular orbit of radius r about a planet of mass m. All we know is the satellite's orbital frequency ω. What can we learn about r and m?

In a circular orbit, we know that the centripetal force is provided by gravity, so we have (where μ is the mass of the satellite):

$$\frac{\mu v^2}{r} = \frac{Gm\mu}{r^2} \tag{1}$$

The linear speed v of the satellite is given by

$$v = \omega r \tag{2}$$

so we have

$$\omega^2 = \frac{Gm}{r^3} \tag{3}$$

That is, we can determine only the combination m/r^3 if all we know is ω. MTW refer to this as the 'Kepler density', since it has the units of a density (mass over volume), although the mass is spread over a sphere with a radius equal to that of the orbit (and we're missing the $\frac{4}{3}\pi$ factor from the volume of the sphere).