FOUR-MOMENTUM OF PHOTONS

The four-momentum is defined for a particle with a rest mass $m$ as

$$\mathbf{p} = \gamma m [1, v_x, v_y, v_z]$$

where $\gamma = 1/\sqrt{1-v^2}$.

For particles such as photons that have no rest mass, this formula obviously doesn’t work. However, the first component of $\mathbf{p}$ is taken as the particle’s energy, so if we make that identification for photons, we get

$$\mathbf{p} = E [1, v_x, v_y, v_z]$$

For any massless particle, $v = \sqrt{v_x^2 + v_y^2 + v_z^2} = 1$, so $\mathbf{p} \cdot \mathbf{p} = E^2 (v_x^2 + v_y^2 + v_z^2 - 1) = 0$, making $\mathbf{p}$ a null vector.

As an example, suppose we have a positive pion $\pi^+$ (rest mass 140 MeV) at rest that decays into an antimuon $\mu^+$ (rest mass 106 MeV) and a neutrino. Avoiding any controversy over whether or not the neutrino has mass, we’ll just assume it’s massless so its momentum is (assuming it travels along the $+x$ axis):

$$\mathbf{p}_\nu = E_\nu [1, 1, 0, 0]$$

For the muon we have

$$\mathbf{p}_\mu = E_\mu [1, v, 0, 0]$$

where $E_\mu = 106\gamma$ MeV.

From conservation of momentum we get from the energy component

$$E_\nu + E_\mu = E_\pi$$

$$E_\nu + 106\gamma = 140$$

From the second component we get
\[ E_\nu + vE_\mu = 0 \] \hspace{1cm} (7)
\[ E_\nu + 106v\gamma = 0 \] \hspace{1cm} (8)

Subtracting these two equations we get

\[ 106\gamma (1 - v) = 140 \] \hspace{1cm} (9)
\[ \frac{1 - v}{\sqrt{1 - v^2}} = 1.32 \] \hspace{1cm} (10)
\[ 1 - 2v + v^2 = 1.744 - 1.744v^2 \] \hspace{1cm} (11)

This quadratic has only one acceptable solution (there is also \( v = 1 \) which is spurious since it makes \( \gamma \) infinite), which is \( v = -0.271 \). This gives \( \gamma = 1.0389 \), from which we get the energies:

\[ E_\nu = 140 - 106\gamma \] \hspace{1cm} (12)
\[ = 29.88 \text{ MeV} \] \hspace{1cm} (13)
\[ E_\mu = 110.12 \text{ MeV} \] \hspace{1cm} (14)

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