FOUR-MOMENTUM CONSERVATION IN ELECTRON-POSITRON ANNIHILATION

When a positron and an electron colliding have the same energy $E$.

\[ E = \gamma m \]

Now suppose we look at the problem in the electron’s frame. Before the collision, the situation is the same as the one we analyzed earlier, so the momentum before the collision is

\[ \mathbf{p}_1' = 2m\gamma^2[1, -v, 0, 0] \]
frame to find their values in the electron’s frame. The transformations for the $t$ and $x$ components are

$$p'_t = \gamma p_t - v \gamma p^x \quad (8)$$

$$p'_x = -v \gamma p_t + \gamma p^x \quad (9)$$

Applying this to $p_2$ we get

$$p'_2 = \gamma E [1 - v, 1 - v, 0, 0] + \gamma E [1 + v, -1 - v, 0, 0] \quad (10)$$

$$= 2 \gamma E [1, -v, 0, 0] \quad (11)$$

Since $E = \gamma m$, we see that $p'_1 = p'_2$ so four-momentum is conserved in the electron’s frame as well.

Note that we could have just applied the Lorentz transformation to the final form of $p_2$: we didn’t need to work out the transformations on each photon separately. However it’s interesting to see how the two photons transform. The energy of the photon travelling to the right is reduced by a factor of $\gamma (1 - v) = \sqrt{(1 - v)/(1 + v)}$, while that of the photon moving to the left is increased by a factor of $\gamma (1 + v) = \sqrt{(1 + v)/(1 - v)}$. Since the speeds of the photons remain unchanged (the speed of light), this change in energy is reflected in a change of their wavelengths. This is of course the Doppler effect.

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