We saw in the graph of the brightness of a background source when lensed by a MACHO that the main brightening occurs for times such that \(-0.5 < t/t_E < +0.5\), so the MACHO moves about the radius of the Einstein ring \(\theta_E\):

\[
\theta_E \equiv \sqrt{\frac{4GM}{D_{LS}D_LD_S}} \quad (1)
\]

If we consider a solar mass MACHO, then \(GM = 1.477 \text{ km} = 1.56 \times 10^{-13} \text{ ly}\). If we take the distance of the MACHO as 30,000 light years and the distance of the source as twice that, then \(D_L = D_{LS} = 3 \times 10^4 \text{ ly}\) and \(D_S = 6 \times 10^4 \text{ ly}\). The angle is then

\[
\theta_E = 3.226 \times 10^{-9} \text{ radians} \quad (2)
\]

At a distance of \(D_L\), this corresponds to an actual distance of

\[
d_E = 9.15 \times 10^8 \text{ km} \quad (3)
\]

If the MACHO has a transverse speed of 200 km/s it will take it about 53 days to move this distance.

The MACHO in Moore’s Fig. 13.6 takes about 10 days to travel an angle of \(\theta_{E0}\) so if the distances and speed are the same as in this example, the mass \(M_0\) of the MACHO is found from

\[
\frac{\theta_{E0}}{\theta_E} = \frac{10}{53} = \sqrt{\frac{M_0}{M}} \quad (4)
\]

This gives the MACHO a mass of around 0.04 solar masses, which could fit if the MACHO is a brown dwarf star, as some are believed to be.