BLANK HOLE EVAPORATION: REMNANTS OF THE BIG BANG

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We’ve seen that through radiation, a black hole can eventually evaporate in a time $t_0$:

$$ t_0 = 2.0903 \times 10^{67} \left( \frac{M}{M_s} \right)^{3/2} \text{ years} \tag{1} $$

where $M_s$ is the solar mass. Clearly if we’re going to observe black hole evaporation, its mass $M$ must be considerably less than the sun’s. Suppose black holes were formed during the big bang at $13.7 \times 10^9$ years ago. If the black hole is just evaporating now, its mass would have been:

$$ M = M_s \left( \frac{13.7 \times 10^9}{2.0903 \times 10^{67}} \right)^{1/3} $$

$$ = 8.69 \times 10^{-20} M_s \tag{2} $$

$$ = (8.69 \times 10^{-20}) \left( 1.989 \times 10^{30} \right) \text{ kg} \tag{3} $$

$$ = 1.728 \times 10^{11} \text{ kg} \tag{4} $$

To put this in perspective, this is equivalent to an asteroid, with a typical rocky density of $\rho = 5000 \text{ kg m}^{-3}$, with a radius of

$$ R = \left( \frac{3M}{4\pi\rho} \right)^{1/3} \tag{6} $$

$$ = 202 \text{ m} \tag{7} $$

To see how much energy is released in the final second of the black hole’s life, we can start with the equation we had earlier from the Stefan-Boltzmann relation:

$$ M^2 dM = -\frac{\sigma h^4}{256\pi^3 k_B^4 G^2} dt \tag{8} $$
If we integrate this from $t = 0$ to a time $t_1$ one second before the present, at which time the black hole’s remaining mass is $M_1$, then

$$\int_{M_0}^{M_1} M^2 dM = -\frac{\sigma \hbar^4}{256 \pi^3 k_B^4 G^2} \int_0^{t_1} dt$$  \hspace{1cm} (9)$$

$$\frac{1}{3} (M_1^3 - M_0^3) = -\frac{\sigma \hbar^4}{256 \pi^3 k_B^4 G^2} t_1$$  \hspace{1cm} (10)$$

$$M_1 = \left[ M_0^3 - \frac{3\sigma \hbar^4}{256 \pi^3 k_B^4 G^2} t_1 \right]^{1/3}$$  \hspace{1cm} (11)$$

However, $M_0^3$ is just $\frac{3\sigma \hbar^4}{256 \pi^3 k_B^4 G^2} t_p$, where $t_p$ is the present time, so that $t_p - t_1 = 1$ s. Therefore the mass remaining 1 second before complete evaporation is:

$$M_1 = \left[ \frac{3\sigma \hbar^4}{256 \pi^3 k_B^4 G^2} (t_p - t_1) \right]^{1/3}$$  \hspace{1cm} (12)$$

$$= 341.38 (t_p - t_1)^{1/3}$$  \hspace{1cm} (13)$$

In GR units, a time of 1 second is $3 \times 10^8$ m so we get for the mass

$$M_1 = 2.28 \times 10^5 \text{ kg}$$  \hspace{1cm} (14)$$

This is equivalent to

$$E = M_1 c^2 = 2.06 \times 10^{22} \text{ J}$$  \hspace{1cm} (15)$$

which is about 500,000 times the energy released in an atomic bomb blast.

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