RIEMANN TENSOR IN 2-D CURVED SPACE

As usual, we need the Christoffel symbols, which we can get by comparing the two forms of the geodesic equation. These equations are

\[ g_{ai} \dddot{x}^i + \left( \partial_i g_{aj} - \frac{1}{2} \partial_a g_{ij} \right) \dddot{x}^j \dddot{x}^i = 0 \]  
\[ \dddot{x}^m + \Gamma^m_{ij} \dddot{x}^j \dddot{x}^i = 0 \]

The metric is

\[ ds^2 = \frac{dp^2}{1 - kp^2} + p^2 dq^2 \]

so \( g_{pp} = 1 / (1 - kp^2) \) and \( g_{qq} = p^2 \). For the two coordinates, this gives us

\[ \frac{1}{1 - kp^2} \dddot{p} + \frac{k p}{(1 - kp^2)^2} \dddot{p}^2 - p \dddot{q}^2 = 0 \]  
\[ p^2 \dddot{q} + 2p \dddot{q} = 0 \]

Dividing through by the coefficient of the second derivative in each case gives:

\[ \dddot{p} + \frac{k p}{1 - kp^2} p^2 - p (1 - kp^2) \dot{\dot{q}} = 0 \]  
\[ \dddot{q} + \frac{2 \dddot{q}}{p} = 0 \]

Comparing with we get
The only independent Riemann tensor component in 2-d is \( R_{qpq} \):

\[
R_{qpq} = \partial_p \Gamma_{qq}^p - \partial_q \Gamma_{pq}^p + \Gamma_{kp}^p \Gamma_{qq}^k - \Gamma_{qk}^p \Gamma_{pq}^k
\]

\[
= - (1 - 3kp^2) - kp^2 + (1 - kp^2)
\]

\[
= kp^2
\]

Any non-zero component indicates that the space is curved, so this metric represents a curved space.