METRIC TENSOR AS A STRESS-ENERGY TENSOR

A curious possibility for the stress-energy tensor is

\[ T^{ij} = -\Lambda g^{ij} \]  

(1)

where \( \Lambda \) is a positive constant and \( g^{ij} \) is any metric tensor. In a local inertial frame (LIF), \( g^{ij} = \eta^{ij} \) and

\[ T^{ij} = \Lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \]  

(2)

Note that this tensor satisfies the energy conservation equation \( \nabla_i T^{ij} = 0 \) as the covariant derivative \( \nabla_i g^{ij} \) is always zero.

Comparing this with the form of \( T^{ij} \) for a perfect fluid in its LIF

\[ T^{ij} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{bmatrix} \]  

(3)

we see that the energy density is \( \rho = \Lambda > 0 \) and the pressure is \( P = -\Lambda < 0 \). Such a tensor cannot arise from a perfect fluid because for such a fluid, for example

\[ T^{xxx} = \int \int \int N(p) \left( \frac{p^x}{p^t} \right)^2 \frac{dp^x dp^y dp^z}{p^t V} \]  

(4)

The integrand is intrinsically non-negative because \( N(p) \) is the number of particles in volume \( V \) with a magnitude of momentum \( p \) and must be non-negative. The component \( p^t = \gamma m \) for massive particles or \( p^t = E \) for photons, but in either case it too is positive. The numerator \( (p^x)^2 \), being a square, is also non-negative. Thus for a perfect fluid in its LIF, \( T^{xxx} \geq 0 \).
Although this tensor cannot be that of a perfect fluid, it does play a role in general relativity as we’ll hopefully see a bit later.

PINGBACKS

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