The general relativistic generalization of Newton’s law of gravity is

\[ G^{ij} + \Lambda g^{ij} = \kappa T^{ij} \]  

(1)

where the Einstein tensor is defined in terms of the Ricci tensor and the curvature scalar as

\[ G^{ij} \equiv R^{ij} - \frac{1}{2} g^{ij} R \]  

(2)

We can write this in a different form that is sometimes easier to use in calculations. Eliminating \( G^{ij} \) we have

\[ R^{ij} - \frac{1}{2} g^{ij} R + \Lambda g^{ij} = \kappa T^{ij} \]  

(3)

Multiplying both sides by \( g^{ij} \) we get

\[ g^{ij} R^{ij} - \frac{1}{2} g^{ij} g^{jk} R + \Lambda g_{ij} g^{ij} = \kappa g_{ij} T^{ij} \]  

(4)

Because the tensor \( g_{ij} \) is the inverse of \( g^{ij} \), their product gives the identity matrix of rank 4 (this can be seen by doing the calculation in a local inertial frame where \( g_{ij} = \eta_{ij} \) and noting that since it’s a tensor equation, it’s valid in all coordinate systems). That is

\[ g_{ij} g^{jk} = \delta^k_i \]  

(5)

so if we contract the \( \delta^k_i \) tensor we just sum up its diagonal elements and since these are all 1 (and there are four rows), we get

\[ \delta^k_k = 4 \]  

(6)

Returning to (4) we get

\[ g_{ij} R^{ij} - 2 R + 4 \Lambda = \kappa g_{ij} T^{ij} \]  

(7)
EINSTEIN EQUATION: ALTERNATIVE FORM

Since the curvature scalar is given by

\[ R \equiv g_{ij} R^{ij} \quad (8) \]

and the stress-energy scalar is

\[ T \equiv g_{ij} T^{ij} \quad (9) \]

we get

\[ -R + 4\Lambda = \kappa T \quad (10) \]

Multiplying this by \(-\frac{1}{2} g^{ij}\) and subtracting from 3 we have

\[ R^{ij} - \Lambda g^{ij} = \kappa \left( T^{ij} - \frac{1}{2} g^{ij} T \right) \quad (11) \]

Isolating the Ricci tensor gives us the alternative form of the Einstein equation:

\[ R^{ij} = \kappa \left( T^{ij} - \frac{1}{2} g^{ij} T \right) + \Lambda g^{ij} \quad (12) \]