PV DIAGRAMS: A DIATOMIC IDEAL GAS UNDERGOES A
RECTANGULAR CYCLE

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Reference: Daniel V. Schroeder, An Introduction to Thermal Physics,
(Addison-Wesley, 2000) - Problem 1.34.
Post date: 6 Jul 2015

Here is a slightly more complex example of using a PV diagram to deduce some facts about heat and work flows into or out of an ideal gas. This time, the path is a rectangle starting at $P_1$ and $V_1$. On side A, the pressure is increased to $P_2$ while the volume is held constant. Then on side B, the volume is increased to $V_2$ while the pressure is constant at $P_2$. On side C, the pressure is decreased back to $P_1$ with the volume constant at $V_2$. Finally, on side D, the volume is decreased back to $V_1$ with the pressure constant at $P_1$.

The gas in this case is diatomic, but the temperature is low enough that only the translational and rotational degrees of freedom are excited; vibrational modes are frozen out. This means that the energy of the gas is

$$U = \frac{5}{2} N k T = \frac{5}{2} PV \quad (1)$$

The work done on any path is

$$W = - \int_{V_i}^{V_f} P(V) \, dV \quad (2)$$

and the heat is obtained from conservation of energy

$$Q = \Delta U - W \quad (3)$$

We can do similar calculations to our earlier example of the triangular path to get

<table>
<thead>
<tr>
<th>Side</th>
<th>$W$</th>
<th>$\Delta U$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>$\frac{5}{2} V_1 (P_2 - P_1)$</td>
<td>$\frac{5}{2} V_1 (P_2 - P_1)$</td>
</tr>
<tr>
<td>B</td>
<td>$-P_2 (V_2 - V_1)$</td>
<td>$\frac{5}{2} P_2 (V_2 - V_1)$</td>
<td>$-\frac{5}{2} P_2 (V_2 - V_1)$</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>$-\frac{5}{2} V_2 (P_2 - P_1)$</td>
<td>$-\frac{5}{2} V_2 (P_2 - P_1)$</td>
</tr>
<tr>
<td>D</td>
<td>$P_1 (V_2 - V_1)$</td>
<td>$-\frac{5}{2} P_1 (V_2 - V_1)$</td>
<td>$-\frac{5}{2} P_1 (V_2 - V_1)$</td>
</tr>
<tr>
<td>Total</td>
<td>$-(P_2 - P_1) (V_2 - V_1)$</td>
<td>0</td>
<td>$(P_2 - P_1) (V_2 - V_1)$</td>
</tr>
</tbody>
</table>
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Along side A, no work is done, but heat is added to the gas to increase the pressure. Along side B, the gas expands doing work on the piston, but heat must be added to achieve this. Along side C, again no work is done and the gas gives off heat. Along side D, work must be done on the gas to compress it, and in the process the gas gives off heat.

PINGBACKS

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