The heat capacity of a star was estimated using the virial theorem as

\[ C_V = -\frac{3}{2} Nk \]  

where \( N \) is the number of particles (typically dissociated protons and electrons) in the star. A negative heat capacity is typical of gravitationally bound systems.

We can use this to work out the entropy from the formula

\[ S = \int \frac{C_V(T)}{T} dT \]

\[ = -\frac{3}{2} Nk \ln T + f(N, V) \]

where \( f \) is some function that depends on \( N \) and the volume \( V \), but not on \( T \).

The total energy of a gravitationally bound system is negative and, from the virial theorem, we have

\[ U = -K = -\frac{3}{2} NkT \]

where \( K \) is the kinetic energy and the formula is obtained from the equipartition theorem. Thus the entropy can be written in terms of the energy as

\[ S = -\frac{3}{2} Nk \ln \left| \frac{2U}{3Nk} \right| + f(N, V) \]

We can incorporate everything inside the logarithm except for \( U \) into the function \( f \) (call the new function \( g \), say), so that

\[ S = -\frac{3}{2} Nk \ln |U| + g(N, V) \]
The general shape of this curve is like this (units on the axes are arbitrary as I’m just trying to show the shape of the graph):

![Graph illustration]

The graph is concave upwards, which is typical of systems with negative heat capacity as we discussed earlier. For sufficiently low (large negative) values of $U$, the graph would go negative, but I would guess that the temperature at that point would be higher than anything found in real stars so that would never happen. Note that as $U \to 0$, $S$ effectively becomes infinite. At $U = 0$, however, the system becomes gravitationally unbound, so the particles would presumably then be able to wander over the entire universe, giving them an infinite number of possible states.