THERMODYNAMIC PROPERTIES OF A 2-DIM IDEAL GAS

We now revisit the 2-d ideal gas for which the Sackur-Tetrode equation is

\[ S = Nk \left[ \ln \frac{2\pi mAU}{(hN)^2} + 2 \right] \]  

(1)

where \( A \) is the area occupied by the gas, \( N \) is the number of molecules, each of mass \( m \), and \( U \) is the total energy. We can work out the temperature, pressure and chemical potential by applying the thermodynamic identity adapted for 2 dimensions (by replacing the volume \( V \) by the area \( A \)):

\[ dU = TdS - PdA + \mu dN \]  

(2)

The temperature is determined from the entropy as

\[ \frac{1}{T} = \left( \frac{\partial S}{\partial U} \right)_{A,N} \]  

(3)

\[ = Nk \frac{(hN)^2}{2\pi mA} \frac{2\pi mA}{(hN)^2} \]  

(4)

\[ = \frac{Nk}{U} \]  

(5)

This just gives us the formula from the equipartition theorem for a system with 2 degrees of freedom:

\[ U = \frac{2}{2} NkT = NkT \]  

(6)

The pressure can be obtained from


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\[ P = T \left( \frac{\partial S}{\partial A} \right)_{U,N} \]  
(7)

\[ = N k \left( \frac{(hN)^2}{2 \pi m U} \right) \]  
(8)

\[ = \frac{N k T}{A} \]  
(9)

This is just the 2-dim analogue of the ideal gas law:

\[ PA = NkT \]  
(10)

Finally, chemical potential is defined in terms of the entropy as

\[ \mu \equiv -T \left( \frac{\partial S}{\partial N} \right)_{U,A} \]  
(11)

\[ = -kT \left[ \ln \left( \frac{2 \pi m A U}{(hN)^2} \right) + 2 \right] - N k T \left( -\frac{2}{N} \right) \]  
(12)

\[ = -kT \ln \left( \frac{2 \pi m A U}{(hN)^2} \right) \]  
(13)

\[ = -kT \ln \left( \frac{A 2 \pi m k T}{N} \right) \]  
(14)

We can compare this to the chemical potential for a 3-d ideal gas

\[ \mu = -kT \ln \left[ \frac{V}{N} \left( \frac{2 \pi m k T}{h^2} \right)^{3/2} \right] \]  
(15)

The only differences are the replacement of \( V \) by \( A \) and the change in the exponent inside the logarithm from \( \frac{3}{2} \) to 1. The latter arises from the derivation of the multiplicity, where the exponent depends on the number of degrees of freedom in the system. For a 3-d gas, there are \( 3N \) degrees of freedom, while for a 2-d gas, there are \( 2N \). Thus the exponent in the 2-d case is \( \frac{2}{3} \) that in the 3-d case. [You’d need to follow through the derivation in detail to see the difference, but basically that’s where it comes from.]