AIR CONDITIONERS

An air conditioner is an example of a refrigerator in which the cold reservoir is the room to be cooled and the hot reservoir is the outside atmosphere. On a hot day, the rate at which heat leaks into an air conditioned room from the outside is roughly proportional to the temperature difference $T_h - T_c$ between the outside and inside. In that case, the work done to remove an amount $Q_c$ of heat in time $\Delta t$ is

$$W = Q_h - Q_c = Q_h - K (T_h - T_c) \tag{1}$$

where $Q_h$ is the heat expelled to the outside and $K$ is a constant.

In an ideal refrigerator (e.g. one working on a reversed Carnot cycle) the entropy gained in absorbing $Q_c$ is equal to the entropy lost in expelling $Q_h$, so

$$\frac{Q_c}{T_c} = \frac{Q_h}{T_h} \tag{2}$$

$$Q_h = \frac{T_h}{T_c} Q_c \tag{3}$$

$$= K \frac{T_h}{T_c} (T_h - T_c) \tag{4}$$

The work required to maintain a temperature of $T_c$ is therefore

$$W = K \frac{T_h}{T_c} (T_h - T_c) - K (T_h - T_c) \tag{5}$$

$$= \frac{K}{T_c} (T_h - T_c)^2 \tag{6}$$

Thus lowering the inside temperature by a small amount can have a large effect on the work required to maintain this temperature, and thus on the cost of running the air conditioner. For example, suppose the outside temperature is $30^\circ C = 303 \text{ K}$ and the inside temperature is $22^\circ C = 295 \text{ K}$. If
we wish to lower the inside temperature by only one degree, the extra work required is

\[
\frac{W_{294}}{W_{295}} = \frac{295 \times 9^2}{294 \times 8^2} = 1.27
\]  

(7)

We need to use 27% more power to achieve a single degree more cooling. This is one reason why it is much more economical to bear with a slightly higher indoor temperature on a hot day.

PINGBACKS

Pingback: Heat pumps