An absorption refrigerator replaces the electrical work $W$ with heat $Q_f$ from burning a fuel such as propane. [It may seem bizarre that we can cool things by applying a flame, but these refrigerators do actually exist, and are used in places where electricity isn’t available. The actual physics and chemistry used is fairly involved, so I won’t go into it here.] As in a regular refrigerator, the goal is to use this input energy to extract heat $Q_c$ from inside the refrigerator and expel the waste heat $Q_r$ into the room.

Since the fuel is the cost in operating the refrigerator and the heat extracted is the benefit, the coefficient of performance here is

$$\text{COP} = \frac{Q_c}{Q_f}$$

(1)

From conservation of energy, we have

$$Q_r = Q_f + Q_c$$

(2)

which gives a COP of

$$\text{COP} = \frac{Q_c}{Q_r - Q_c} = \frac{1}{Q_r/Q_c - 1}$$

(3)

This is the same formula as for a regular refrigerator, with $Q_h$ replaced by $Q_r$. If $Q_r/Q_c < 2$, the COP can be greater than 1.

The second law says that the entropy absorbed by the working substance must be no greater than the entropy expelled into the room. That is

$$\frac{Q_f}{T_f} + \frac{Q_c}{T_c} \leq \frac{Q_r}{T_r}$$

(4)

The optimum performance occurs when we have equality so, using 2 we have

$$\frac{Q_r - Q_c}{T_f} + \frac{Q_c}{T_c} = \frac{Q_r}{T_r}$$

(5)

Solving for $Q_r/Q_c$ we have
\[
\frac{Q_r}{Q_c} = \left( \frac{1}{T_f} - \frac{1}{T_c} \right) \left( \frac{1}{T_f} - \frac{1}{T_r} \right)^{-1} \tag{6} \\
= \frac{T_r (T_c - T_f)}{T_c (T_r - T_f)} \tag{7}
\]

Substituting into (3) we have

\[
\text{COP}_{\text{max}} = \left( \frac{T_r (T_c - T_f)}{T_c (T_r - T_f)} - 1 \right)^{-1} \tag{8} \\
= \frac{T_c (T_r - T_f)}{T_r (T_c - T_f) - T_c (T_r - T_f)} \tag{9} \\
= \frac{T_c (T_f - T_r)}{T_f (T_r - T_c)} \tag{10}
\]

The COP decreases as \( T_r - T_c \) increases as we’d expect: the larger the temperature difference between inside and outside, the larger the energy required to run the refrigerator. If we write the COP as

\[
\text{COP}_{\text{max}} = \frac{T_c}{T_r - T_c} \left( 1 - \frac{T_r}{T_f} \right) \tag{11}
\]

we see that increasing the flame’s temperature also increases the COP. This gives the rather counter-intuitive result that the hotter the flame used to power the fridge, the better is its performance.