COUPLING OF PROTON’S MAGNETIC MOMENT TO EXTERNAL FIELD

We’ve dealt with the Zeeman effect in a lot of detail before, but Shankar deals with it using the approximation of neglecting the coupling of the proton’s magnetic moment to the external magnetic field $B$. Using classical arguments, we can see why this is a reasonable approximation.

The proton, like the electron, has both orbital and spin angular momentum. The proton’s spin is $\frac{h}{2}$, the same as the electron, so its spin magnetic moment is given by

$$\mu_{ps} = \frac{q}{2M} \frac{\hbar}{c}$$  \hspace{1cm} (1)

where $M$ is the proton’s mass. Since (apart from the sign) the proton and electron have the same charge $q$ and spin, the equivalent formula for the electron is

$$\mu_{es} = \frac{q}{2m} \frac{\hbar}{c}$$  \hspace{1cm} (2)

where $m$ is the electron mass. Thus

$$\mu_{ps} = \frac{m}{M} \mu_{es}$$  \hspace{1cm} (3)

so that the proton’s spin magnetic moment is about $\frac{1}{1836}$ times that of the electron.

For the orbital magnetic moment, we can consider a classical system in which the electron and proton are orbiting about their centre of mass. The period $T$ of the orbit is the same for both particles, and the radius of each orbit is
COUPLING OF PROTON’S MAGNETIC MOMENT TO EXTERNAL FIELD

\[ r_p = \frac{m}{m + M} r \approx \frac{m}{M} r \]\( (4) \)

\[ r_e = \frac{M}{m + M} r \approx r \]\( (5) \)

where \( r \) is the distance between the two particles. The orbital magnetic moment can be written as

\[ \mu_i = \frac{qv_i r_i}{2c} \]\( (6) \)

where the subscript \( i \) is either \( e \) or \( p \). Since the proton moves in a smaller orbit but at the same frequency as the electron, its velocity is smaller. We have

\[ v_p = \frac{2\pi r_p}{T} = \frac{2\pi r}{T} \frac{m}{m + M} \approx \frac{2\pi r}{T} \frac{m}{M} \]\( (7) \)

\[ v_e = \frac{2\pi r_e}{T} = \frac{2\pi r}{T} \frac{M}{m + M} \approx \frac{2\pi r}{T} \]\( (8) \)

Therefore

\[ \mu_p \approx \frac{\pi q r^2}{cT} \left( \frac{m}{M} \right)^2 \]\( (9) \)

\[ \mu_e \approx \frac{\pi q r^2}{cT} \]\( (10) \)

Thus the orbital magnetic moment of the proton is about \( \left( \frac{m}{M} \right)^2 \) times that of the electron.

PINGBACKS
Pingback: Stern-gerlach experiment
Pingback: Hyperfine interaction in hydrogen - a rough calculation