ENERGY CHANGE FOR CHARGED PARTICLE

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References: Kip S. Thorne & Roger D. Blandford, *Modern Classical Physics*, Princeton University Press (2017). Exercise 1.1.

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As a first exercise in doing classical physics geometrically, we consider a particle with charge q and velocity $\boldsymbol{v}$ in an electromagnetic field. Such a particle obeys the Lorentz force law for a combined electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$

$$\boldsymbol{F}=q(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}) \tag{1}$$

The kinetic energy is given by the usual classical formula

$$E=\frac{1}{2}m\boldsymbol{v}^2 \tag{2}$$

The rate of change of the kinetic energy is then

$$\frac{dE}{dt}=\frac{1}{2}m\left(\frac{d\boldsymbol{v}}{dt}\cdot\boldsymbol{v}+\boldsymbol{v}\cdot\frac{d\boldsymbol{v}}{dt}\right) \tag{3}$$

$$=m\boldsymbol{v}\cdot\frac{d\boldsymbol{v}}{dt} \tag{4}$$

Using Newton's force law

$$\boldsymbol{F}=m\boldsymbol{a}=m\frac{d\boldsymbol{v}}{dt} \tag{5}$$

together with 1 we have

$$\frac{dE}{dt}=\boldsymbol{v}\cdot\boldsymbol{F} \tag{6}$$

$$=q(\boldsymbol{v}\cdot\boldsymbol{E}+\boldsymbol{v}\cdot(\boldsymbol{v}\times\boldsymbol{B})) \tag{7}$$

$$=q\boldsymbol{v}\cdot\boldsymbol{E} \tag{8}$$

where the last line follows because $\boldsymbol{v}\times\boldsymbol{B}$ is perpendicular to $\boldsymbol{v}$ so the second dot product on the second line is zero.