TENSOR COMPONENT MANIPULATION RULES

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References: Kip S. Thorne & Roger D. Blandford, *Modern Classical Physics*, Princeton University Press (2017). Exercise 1.3.

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T&B's definition of a tensor is an object which takes a number of vectors as input (the number is the *rank* of the tensor) and outputs a single number. In their eqn 1.4d, the inner product is defined as the output produced when a vector (considered as a rank-1 tensor) takes another vector as input:

$$\boldsymbol{A}\cdot\boldsymbol{C}=\boldsymbol{A}(\boldsymbol{C}) \tag{1}$$

Using eqn 1.9b, we can write a vector $\boldsymbol{A}$ in terms of its components in a Cartesian coordinate system with basis vectors $\boldsymbol{e}_j$:

$$\boldsymbol{A}=A_j\boldsymbol{e}_j \tag{2}$$

where there is an implied sum over $j=1,2,3$. Since the basis vectors $\boldsymbol{e}_j$ are orthonormal, we have

$$\boldsymbol{e}_i\cdot\boldsymbol{e}_j=\delta_{ij} \tag{3}$$

and, using the above, we have

$$\boldsymbol{A}\cdot\boldsymbol{e}_j=A_i\boldsymbol{e}_i\cdot\boldsymbol{e}_j=A_i\delta_{ij}=A_j \tag{4}$$

Ex 1.3(a)

The scalar product of two arbitrary vectors is then

$$\begin{aligned}\boldsymbol{A}\cdot\boldsymbol{B}&=(A_i\boldsymbol{e}_i)\cdot(B_j\boldsymbol{e}_j) &(5)\\&=A_iB_j(\boldsymbol{e}_i\cdot\boldsymbol{e}_j) &(6)\\&=A_iB_j\delta_{ij} &(7)\\&=A_iB_i &(8)\end{aligned}$$

This is the familiar expression for a scalar product expressed in terms of components.

Ex. 1.3(b) For a general rank-3 tensor, we have T&B's eqn 1.9d:

$$\mathsf{T} = T_{ijk}\boldsymbol{e}_i \otimes \boldsymbol{e}_j \otimes \boldsymbol{e}_k \tag{9}$$

This gives the components of the tensor for a given Cartesian coordinate system.

A rank-3 tensor has 3 slots for inputting vectors and, using T&B's eqn 1.5a, we have

$$\mathsf{T}(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C}) = T_{ijk}\boldsymbol{e}_i \otimes \boldsymbol{e}_j \otimes \boldsymbol{e}_k(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C}) \tag{10}$$

$$= T_{ijk}(\boldsymbol{e}_i \cdot \boldsymbol{A})(\boldsymbol{e}_j \cdot \boldsymbol{B})(\boldsymbol{e}_k \cdot \boldsymbol{C}) \tag{11}$$

$$= T_{ijk}A_iB_jC_k \tag{12}$$

This is similar to the treatment of one-forms and vectors in 4 dimensions that we met earlier, except here we're considering only 3-dim flat space.

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