NUMERICS OF TENSOR COMPONENT MANIPULATIONS

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References: Kip S. Thorne & Roger D. Blandford, *Modern Classical Physics*, Princeton University Press (2017). Exercise 1.4.

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This exercise is a bit of practice in dealing with tensor components using specific numerical values. We are given a rank-3 tensor $\mathsf{S}(\_,\_,\_)$ and two vectors $\boldsymbol{A}$and $\boldsymbol{B}$. The non-zero components of these objects are

$$\begin{aligned} S_{123} = S_{231} = S_{312} &= +1 \\ A_1 &= 3 \\ B_1 &= 4 \\ B_2 &= 5 \end{aligned} \tag{1}$$

Some of the solution is given in the question, but we'll go through it here for completeness.

First, we have the object

$$\boldsymbol{C} = \mathsf{S}(\boldsymbol{A},\boldsymbol{B},\_) \tag{2}$$

If we had provided 3 vectors as input for S we would produce a single number as output:

$$\mathsf{S}(\boldsymbol{A},\boldsymbol{B},\boldsymbol{F}) = S_{ijk}A_iB_jF_k \tag{3}$$

Since the last slot is empty, the third index (the k index) is free, so the object $\mathsf{S}(\boldsymbol{A},\boldsymbol{B},\_)$ is an object with one index, which is a vector, or rank-1 tensor. In terms of components, this is

$$C_k = S_{ijk}A_iB_j \tag{4}$$

To find the components C_k we just plug in the values 1 and do the sums. Since the only component of $\boldsymbol{A}$ that is non-zero is A_1, the first index i can only be 1. Because B_1 and B_2 are non-zero, j can be 1 or 2. The only non-zero component of S that has $i=1$ is S_{123} so this is the only non-zero contribution to C_k which means $k=3$ so the only non-zero components of $\boldsymbol{C}$ is C_3, and we have

$$C_3 = S_{123} A_1 B_2 = 15 \tag{5}$$

Now we consider the vector $\boldsymbol{D}$ defined by

$$\boldsymbol{D} \equiv \mathsf{S}(\boldsymbol{A}, \_, \boldsymbol{B}) \tag{6}$$

In this case, the middle slot is free, so we have

$$D_j = S_{ijk} A_i B_k \tag{7}$$

Again, we must have $i = 1$ and $k = 1$ or 2. However, there are no non-zero components of S that satisfy these requirements, so all components of $\boldsymbol{D}$ are zero.

Now we consider

$$\mathsf{W} = \boldsymbol{A} \otimes \boldsymbol{B} \tag{8}$$

In components,

$$\mathsf{W} = (A_i \boldsymbol{e}_i) \otimes (B_j \boldsymbol{e}_j) \tag{9}$$

$$= A_i B_j \boldsymbol{e}_i \otimes \boldsymbol{e}_j \tag{10}$$

To find the components of W, we insert the basis vectors into the slots of W, so we have T&B's eqn 1.5a:

$$W_{k\ell} = \mathsf{W}(\boldsymbol{e}_k, \boldsymbol{e}_\ell) \tag{11}$$

$$= A_i B_j \boldsymbol{e}_i \otimes \boldsymbol{e}_j (\boldsymbol{e}_k, \boldsymbol{e}_\ell) \tag{12}$$

$$= A_i B_j (\boldsymbol{e}_i \cdot \boldsymbol{e}_k)(\boldsymbol{e}_j \cdot \boldsymbol{e}_\ell) \tag{13}$$

$$= A_i B_j \delta_{ik} \delta_{j\ell} \tag{14}$$

$$= A_k B_\ell \tag{15}$$

Consulting the values 1, the only non-zero components of W are

$$\begin{aligned} W_{11} &= A_1 B_1 = 12 \\ W_{12} &= A_1 B_2 = 15 \end{aligned} \tag{16}$$

PINGBACKS

Pingback: Tensor slot-naming notation