# GEODESICS ON A SPHERE: AN ANT SEARCHING FOR HONEY

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References: Anthony Zee, *Einstein Gravity in a Nutshell*, (Princeton University Press, 2013) - Prologue, problems 2-3.

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Snell's law is derived by seeking the shortest travel time for a light ray between a point $A$ in a medium with index of refraction $n_A$ and another point $B$ in a medium with a different index of refraction $n_B$. In general relativity, light follows a geodesic curve as it travels through curved spacetime. Suppose we look at the 2-d curved surface of a hemispherical bowl with its axis equal to the vertical $z$ axis. On the outside of the bowl there is an ant which sees a drop of honey on the inside of the bowl. The honey is located directly opposite the ant (that is, a line from the ant to the honey is perpendicular to the $z$ axis and intersects the $z$ axis). What is the ant's shortest route to the honey?

We know that the shortest distance between two points on the surface of a sphere is an arc from a great circle (that is, a circle with a radius equal to that of the sphere). By analogy with the derivation of Snell's law, what we want to find is the point on the rim of the hemisphere where the ant crosses from the outside to the inside. The path from the ant's initial location to the crossing point lies on a great circle, as does the path on the inside of the bowl from the crossing point to the honey. The key point is that, once the ant reaches the crossing point, the distance it has to travel on the inside is the same as if the ant were on a complete sphere and the honey was located diametrically opposite the ant's initial position on the outside.

In Fig. 1, the green dot is the ant and the lower yellow dot is the honey. The ant would have to travel the same distance to get to the honey if the honey were at the upper yellow dot on the outside of the sphere, since all the angles $\theta$ are equal. Thus the ant must travel a distance equal to half the circumference of the sphere to get to the honey, no matter where the ant and honey start off (provided that the ant and honey are directly opposite each other). Thus the distance travelled is always $\pi R$ and since all great circles that pass through the green dot also pass through the upper yellow dot, it doesn't matter what direction the ant sets off; it will always get to the honey after walking the same distance provided that it sticks to a great circle route.

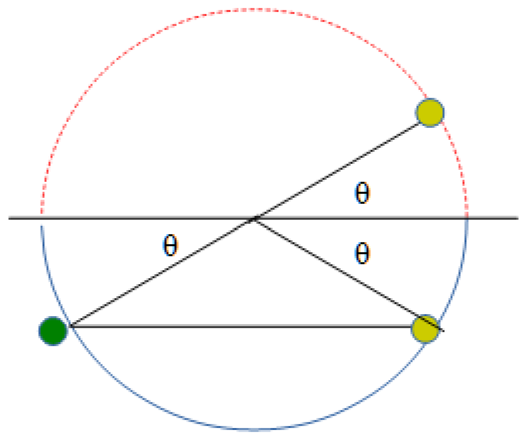


FIGURE 1. An ant searching for honey on a hemisphere.

Now suppose that the ant can walk faster on the outside of the sphere than on the inside. The distance covered is the same, but now the ant wants to maximize the portion of its journey that is on the outside of the bowl. It can do this by always setting off due south (that is, towards the bottom of the bowl) from its starting point.