TENSOR SYMMETRIC IN 2 INDICES

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References: Anthony Zee, *Einstein Gravity in a Nutshell*, (Princeton University Press, 2013) - Chapter I.4, Problem 8.

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In the appendix to Chapter I.4, Zee considers a tensor $G^{ki\cdot j}$ which is symmetric in its first 2 indices, so that

$$G^{ki\cdot j}=G^{ik\cdot j} \tag{1}$$

We then define another tensor in terms of G as

$$H^{k\cdot ij}\equiv G^{ki\cdot j}+G^{kj\cdot i} \tag{2}$$

He states that these two tensors are related by the lemma

$$G^{ki\cdot j}=\frac{1}{2}\left(H^{k\cdot ij}+H^{i\cdot jk}-H^{j\cdot ki}\right) \tag{3}$$

This is easiest to prove by substituting 2 into the RHS. We have

$$H^{k\cdot ij}+H^{i\cdot jk}-H^{j\cdot ki}=G^{ki\cdot j}+G^{kj\cdot i}+G^{ij\cdot k}+G^{ik\cdot j}-G^{jk\cdot i}-G^{ji\cdot k} \tag{4}$$

From 1 we have

$$G^{ki\cdot j}=G^{ik\cdot j} \tag{5}$$

$$G^{kj\cdot i}=G^{jk\cdot i} \tag{6}$$

$$G^{ij\cdot k}=G^{ji\cdot k} \tag{7}$$

Substituting these into 4 we have

$$H^{k\cdot ij}+H^{i\cdot jk}-H^{j\cdot ki}=2G^{ki\cdot j} \tag{8}$$

so

$$\frac{1}{2}\left(H^{k\cdot ij}+H^{i\cdot jk}-H^{j\cdot ki}\right)=G^{ki\cdot j} \tag{9}$$

which proves 3. QED.