# INCREMENTAL DISTANCE WORKS BOTH WAYS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Anthony Zee, *Einstein Gravity in a Nutshell*, (Princeton University Press, 2013) - Chapter I.5, Exercise 1.

Post date: 12 Apr 2020.

The incremental distance from $x$ to $x+dx$, $ds$ is given in terms of the metric tensor $g_{\mu\nu}$ by

$$ds_1^2 = g_{\mu\nu}(x)\,dx^\mu dx^\nu \tag{1}$$

For consistency, this should also be the distance from $x+dx$ to $x$. In this case we have

$$ds_2^2 = g_{\mu\nu}(x+dx)\left(-dx^\mu\right)\left(-dx^\nu\right) \tag{2}$$

The minus signs on the two $dx$s cancel, and we can expand the metric as

$$g_{\mu\nu}(x+dx) = g_{\mu\nu}(x) + \frac{\partial g_{\mu\nu}(x)}{\partial x^\rho}dx^\rho + \ldots \tag{3}$$

If we're considering only terms up to second order in $dx$, the second term on the RHS here can be neglected when inserted back into 2, so we end up with

$$ds_2^2 = g_{\mu\nu}(x)\,dx^\mu dx^\nu = ds_1^2 \tag{4}$$

as required.