LIE DERIVATIVE: PRODUCT RULE

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Post date: 15 Feb 2013.

In the last post we derived the product rule for the Lie derivative. We can verify this rule in the specific case where we have a tensor product $Y^a Z_{bc}$. Using the expression in the last post for the Lie derivative of a general tensor, we have

$$\mathcal{L}_X (Y^a Z_{bc}) = X^d \partial_d (Y^a Z_{bc}) - Y^d Z_{bc} \partial_d X^a + Y^a Z_{dc} \partial_b X^d + Y^a Z_{bd} \partial_c X^d$$

(1)

Multiply the first equation by $Z_{bc}$ and the second by $Y^a$ and add, then compare with the overall derivative above, and we get

$$\mathcal{L}_X (Y^a Z_{bc}) = Z_{bc} \mathcal{L}_X Y^a + Y^a \mathcal{L}_X Z_{bc}$$

(5)